Formule za drugi kolokvij

OSNOVE RAČUNA VJEROJATNOSTI I MATEMATIČKE STATISTIKE

$$w(\mathrm{A}) = \frac{p(\mathrm{A})}{m} \quad 0 \leq p \leq m \quad 0 \leq w \leq 1$$

međusobno isključivi događaji:

$$w(\mathrm{A} + \mathrm{B} + \cdots) = w(\mathrm{A}) + w(\mathrm{B}) + \cdots$$

međusobno nezavisni događaji:

$$w(\mathrm{A}) = w(\mathrm{A}_1) \cdot w(\mathrm{A}_2) \cdot \ldots \cdot w(\mathrm{A}_n)$$

Srednja vrijednost

$$\bar{x} = \sum_i w_i x_i \qquad \sum_i w_i = 1$$

$$\bar{x} = \int x \mathrm{d}w \quad \mathrm{d}w = f(x)\mathrm{d}x \quad \int \mathrm{d}w = 1$$

$$D(x) = \overline{(x - \bar{x})^2} = \overline{x^2} - \bar{x}^2$$

$$\sigma(x) = \sqrt{D(x)} = \sqrt{\overline{x^2} - \bar{x}^2}$$

$$\delta(x) = \frac{\sigma(x)}{\bar{x}}$$

KLASIČNA STATISTIČKA FIZIKA

Maxwellova raspodjela molekula prema translacijskim brzinama

$$f(v_x^2) = \frac{e^{-\frac{m_0 v_x^2}{2kT}}}{\int_{-\infty}^{\infty} e^{-\frac{m_0 v_x^2}{2kT}} \mathrm{d}v_x} = \sqrt{\frac{m_0}{2\pi kT}} e^{-\frac{m_0 v_x^2}{2kT}}$$

$$P(v^2) = \frac{4}{\sqrt{\pi}} \left(\frac{m_0}{2kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{m_0 v^2}{2kT}}$$

$$\int_0^\infty P(v^2)\mathrm{d}v = 1 \qquad \mathrm{d}w = \frac{\mathrm{d}N}{N} = P(v^2)\mathrm{d}v$$

$$v_m = \sqrt{\frac{2kT}{m_0}} \quad \bar{v} = \sqrt{\frac{8kT}{\pi m_0}} \quad v_{sk} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m_0}}$$

Boltzmannova raspodjela

$$\rho(E) = Ce^{-\beta E} \qquad \beta = \frac{1}{kT}$$

$$\mathrm{d}w = \rho(E)\mathrm{d}\phi \qquad \mathrm{d}\phi = \mathrm{d}q_1 \cdots \mathrm{d}q_f \mathrm{d}p_1 \cdots \mathrm{d}p_f$$

$$\bar{x} = \frac{\int x e^{-\beta E} \mathrm{d}\phi}{\int e^{-\beta E} \mathrm{d}\phi}$$

$$U = N\bar{E}$$

Particijska funkcija

$$z = \frac{2s+1}{h^f} \int e^{-\beta E} \mathrm{d}\phi$$

$$\bar{E} = -\left(\frac{\partial}{\partial \beta} \ln z\right)_V = -\frac{\frac{\partial z}{\partial \beta}}{z}$$

$Z = z^N$ klasično

$Z = \frac{1}{N!} z^N$ kvantno

$$F = -kT \ln Z \qquad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

FIZIKA IV

KVANTNA STATISTIČKA FIZIKA

Kvantizacija energijskog spektra

$$N_i = C g_i e^{-\beta E_i}$$

$$\bar{x} = \frac{\sum_i x_i g_i e^{-\beta E_i}}{\sum_i g_i e^{-\beta E_i}}$$

$$z = \sum_i g_i e^{-\beta E_i}$$

$$\bar{E} = \frac{\sum_i E_i g_i e^{-\beta E_i}}{\sum_i g_i e^{-\beta E_i}} = -\frac{\frac{\partial z}{\partial \beta}}{z}$$

Harmonijski oscilator

$$\bar{E} = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$$

$$E_n = \frac{\hbar\omega}{2}(2n+1) \quad n = 0, 1, 2, \ldots$$

Fermioni i bozoni

$$N = \int_0^\infty g(E)\rho(E)\mathrm{d}E$$

$$\bar{x} = \frac{\int_0^\infty x(E)g(E)\rho(E)\mathrm{d}E}{\int_0^\infty g(E)\rho(E)\mathrm{d}E}$$

$$\bar{E} = \frac{\int_0^\infty E g(E)\rho(E)\mathrm{d}E}{\int_0^\infty g(E)\rho(E)\mathrm{d}E} = \frac{1}{N}\int_0^\infty E g(E)\rho(E)\mathrm{d}E$$

$$g(E) = \frac{2s+1}{h^3} 4\pi V m\sqrt{2mE}$$

$$\rho(E) = \frac{1}{e^{\frac{E-\mu}{kT}} \pm 1}$$

$+$ Fermi - Diracova
$-$ Bose - Einsteinova

fermioni na apsolutnoj nuli:

$$\rho(E) = \begin{cases} 1 & E < \mu_0 \\ 0 & E > \mu_0 \end{cases}$$

$$\mu_0 = \frac{\hbar^2}{2m}\left(\frac{6\pi^2}{2s+1}\frac{N}{V}\right)^{\frac{2}{3}}$$

MATEMATIČKI PODSJETNIK

$$\int_{-\infty}^{\infty} e^{-x^2}\mathrm{d}x = \sqrt{\pi}$$

$$\int_0^{\infty} e^{-x^2}\mathrm{d}x = \frac{\sqrt{\pi}}{2}$$

$$I_n = \int_0^\infty x^n\, e^{-ax^2}\mathrm{d}x = \frac{n-1}{2a} I_{n-2}$$

$$I_0 = \int_0^\infty e^{-ax^2}\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$I_1 = \int_0^\infty x e^{-ax^2}\mathrm{d}x = \frac{1}{2a}$$

suma konačnog geometrijskog reda:

$$S_n = a + aq + aq^2 + \cdots + aq^n$$

$$S_n = a\frac{1-q^{n+1}}{1-q}$$

suma beskonačnog geometrijskog reda
$n \to \infty \;\; q < 1$:

$$S = \frac{a}{1-q}$$

Konstante

$k = 1{,}38 \cdot 10^{-23}\ \mathrm{J\,K^{-1}}$

$R = 8{,}314\ \mathrm{J\,mol^{-1}K^{-1}}$

$\sigma = 5{,}67 \cdot 10^{-8}\ \mathrm{W\,m^{-2}K^{-4}}$

$N_A = 6{,}022 \cdot 10^{23}\ \mathrm{mol^{-1}}$

$h = 6{,}626 \cdot 10^{-34}\ \mathrm{J\,s}$

$m_e = 9{,}11 \cdot 10^{-31}\ \mathrm{kg}$

$e = 1{,}6 \cdot 10^{-19}\ \mathrm{C}$